Lesson 1A: Arithmetic Sequences, Explicit Formula

Classwork

Opening Exercise

Mrs. Rosenblatt gave her students what she thought was a very simple task:

What is the next number in the sequence 2, 4, 6, 8, …?

*Cody: I am thinking of a “plus 2 pattern,” so it continues 10, 12, 14, 16, ….*

*Ali: I am thinking a repeating pattern, so it continues 2, 4, 6, 8, 2, 4, 6, 8, ….*

*Suri: I am thinking of the units digits in the multiples of two, so it continues 2, 4, 6, 8, 0, 2, 4, 6, 8, ….*

1. Are each of these valid responses?
2. What is the hundredth number in the sequence in Cody’s scenario? Ali’s? Suri’s?

**Example 1**

Jerry has thought of a pattern that shows powers of two. Here are the first 6 numbers of Jerry’s sequence:

$$1, 2, 4, 8, 16, 32,$$

What are the next three terms in Jerry’s sequence?

A **sequence** is a set of numbers, called the **terms of the sequence** in a specific order.

Consider the sequence that follows a “plus 3” pattern: $4, 7, 10, 13, 16,$ ….

Consider the sequence that follows a “minus 3” pattern: 33, 30, 27, 24, 21,….

|  |
| --- |
| An **arithmetic sequence** is a numerical pattern that increases or decreases at a constant rate called the **common difference** |
| 3, 5, 7, 9, 11, …d = 2 | 33, 29, 25, 21, 17, …d = -4 |

Determine whether each sequence is an arithmetic sequence.

a. -4, -2, 0, 2, … b. 1, 4, 9, 25, ... c. $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{13}{16}, …$ d. -26, -22, -18, -14, …

Find the next three terms in the arithmetic sequence 15, 9, 3, -3, … .

Find the next four terms of the arithmetic sequence 9.5, 11.0, 12.5, 14.0, … .

Each term in an arithmetic sequence can be expressed in terms of the **first term a1** and the **common difference d**.

8, 11, 14, 17, ….

|  |  |  |  |
| --- | --- | --- | --- |
| **Term** | **Symbol** | **In terms of a1 and d** | **Numbers** |
| first term | a1 | a1 | 8 |
| second term | a2 | a1 + d | 8 + 1(3) = |
| third term | a3 | a1 + 2d | 8 + 2(3) = |
| fourth term | a4 | a1 + 3d | 8 + 3(3) = |
| … | … | … | … |
| nth term | an | a1 + (n – 1)d | 8 + (n – 1)(3) |

Write an equation for the nth term of the arithmetic sequence -12, -8, -4, 0, …

**Step 1: Find the common difference.**

**Step 2: Write the equation.**

 **an = a1 + (n – 1)d**

Find the 9th term:

Write an equation for the nth term of the arithmetic sequence 15, 9, 3, -3, … .

**Step 1: Find the common difference.**

**Step 2: Write the equation.**

 **an = a1 + (n – 1)d**

Find the 20th term:

Find the 99th term:

Write an equation for the nth term of the arithmetic sequence 9.5, 11.0, 12.5, 14.0, … .

**Step 1: Find the common difference.**

**Step 2: Write the equation.**

 **an = a1 + (n – 1)d**

Find the 15th term:

Find the 25th term:

Lesson 1B: Arithmetic Sequences, Recursive Formula

In an **arithmetic sequence** each term is found by adding a fixed number called the **common difference (*d*)**, to the previous term. Example: 3, 5, 7, 9,…

The following equation is a recursive definition of an arithmetic sequence:

**an+1= an + d**

|  |  |  |  |
| --- | --- | --- | --- |
| **Term** | **Symbol** | **In terms of a1 and d** | **Numbers** |
| first term | a1 | a1 | 3 |
| second term | a2 | a1 + d | 3 + 2 = |
| third term | a3 | a2 + d | 5 + 2 = |
| fourth term | a4 | a3 + d | 7 + 2 = |
| … | … | … | … |
| nth term | an | an-1 + d | an  |
| nth + 1 term | an+1 | an + d | an+1 |

**Example 1**

**For problems 1-4, list the first five terms of each sequence.**

1. $a\_{n+1}=a\_{n}+6$, where $a\_{1}=11$ for $n\geq 1$

1. $a\_{n+1}=a\_{n}- 6 $ where $a\_{1}=11$ for $n\geq 1$

1. $a\_{n+1}=a\_{n}+4$, where $a\_{1}=3$ for $n\geq 1$

1. Doug accepts a job where his starting salary will be $30,000 per year, and each year he will receive a raise of $3,000.

5. Write the recursive formula for the arithmetic sequence 15, 9, 3, -3, … , where a1 is 15. Find the next three terms.

6. Write the recursive formula for the arithmetic sequence 9.5, 11, 12.5, 14, … , where a1 is 9.5. Find the next four terms.

7. Determine the recursive formula for each of the following arithmetic sequences.

a. 18, 16, 14, 12, …. b. 30, 24, 18, 12, ….

**Example 2**

Consider a sequence given by the formula $a\_{n+1}=a\_{n}-5$ where $a\_{1}=12$.

* 1. List the first five terms of the sequence.
	2. Write an explicit formula.
	3. Find $a\_{6}$ and $a\_{100}$ of the sequence.

1. Graph the first five terms of the arithmetic sequence 3, 5, 7, 9,…

|  |  |
| --- | --- |
| **an** | **an+1 = an + 2** |
| a1 | 3 |
| a2 | 5 |
| a3 | 7 |
| a4 | 9 |
| a5 |  |



2. Graph the first five terms of the arithmetic sequence 3, 6, 9, 12,…

|  |  |
| --- | --- |
| **an** | **an+1 = an + 3** |
| a1 | 3 |
| a2 | 6 |
| a3 | 9 |
| a4 | 12 |
| a5 |  |



3. Graph the first five terms of the arithmetic sequence 15, 9, 3, -3,…



|  |  |
| --- | --- |
| **an** | **an+1 = an – 6**  |
| a1 | 15 |
| a2 | 9 |
| a3 | 3 |
| a4 | -3 |
| a5 |  |

How does this relate to the explicit form?

Write the explicit form **an = a1 + (n – 1)d** for each arithmetic sequence above.

1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

In an **arithmetic sequence** each term is found by adding a fixed number called the **common difference (*d*)**, to the previous term. Example: 1, 3, 5, 7, 9,…

**What is a0?**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Symbol** | **In terms of a1 and d** | **Sequence 1** | **Sequence 2** | **Sequence 3** |
| a0 | a0 | 1 |  |  |
| a1 | a0 + d | 1 + 2 = |  |  |
| a2 | a1 + d | 3 + 2 = |  |  |
| a3 | a2 + d | 5 + 2 = |  |  |
| a4 | a3 + d | 7 + 2 = |  |  |
| … | … | … |  |  |
| an+1 | an + d | an + 2 |  |  |